(Tui ABM) theory

Compound angles

class XI,

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M.C.Q

1. $\sin(A+B) = \sin A\cos B + \cos A\sin B$
2. $\sin(A-B) = \sin A\cos B - \cos A\sin B$
3. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
4. $\cos(A-B) = \cos A\cos B + \sin A\sin B$
5. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$
6. $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$
7. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$
8. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$
9. $\cot(A+B) = \dfrac{\cot A\cot B - 1}{\cot B + \cot A}$
10. $\cot(A-B) = \dfrac{\cot A\cot B + 1}{\cot B - \cot A}$
11. $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}$

11(i)
$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$

12(i) $\sin\left(n\times\frac{\pi}{2} \pm \theta\right) = \pm\cos\theta$

n বিজোড় হলে [illegible]

(ii) $\sin\left(n\times\frac{\pi}{2} \pm \theta\right) = \pm\sin\theta$

n জোড় হলে [illegible]

[illegible]

MCQ

1. (A)
2. (B)
3. $\sin(45^\circ - \theta)$
$= \sin 45\cos\theta - \cos 45\sin\theta$
$= \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta$
$= \frac{1}{\sqrt{2}}(\cos\theta - \sin\theta)$. (D)

4. $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{\pi}{4}-\theta\right)$
$= \dfrac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta} \cdot \dfrac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\cdot\tan\theta}$
$= \dfrac{1+\tan\theta}{1-\tan\theta} \cdot \dfrac{1-\tan\theta}{1+\tan\theta}$
$= 1$ (C)

5. $\cot 2\theta + \tan\theta$
$= \dfrac{\cos 2\theta}{\sin 2\theta} + \dfrac{\sin\theta}{\cos\theta}$
$= \dfrac{\cos 2\theta\cos\theta + \sin 2\theta\sin\theta}{\sin 2\theta\cos\theta}$
$= \dfrac{\cos(2\theta - \theta)}{\sin 2\theta\cos\theta}$
$= \dfrac{\cos\theta}{\sin 2\theta\cos\theta} = \dfrac{1}{\sin 2\theta}$
$= \operatorname{cosec} 2\theta$

1. (iv) $\sec(-75°)$ [illegible]

$= \sec 75°$

$= \dfrac{1}{\cos 75°}$

$= \dfrac{1}{\cos(45°+30°)}$

$= \dfrac{1}{\cos 45° \cos 30° - \sin 45° \sin 30°} = \dfrac{1}{\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}} = \dfrac{1}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

$= \dfrac{2\sqrt{2}}{\sqrt{3}-1}$

$= \dfrac{2\sqrt{2}(\sqrt{3}+1)}{3-1}$

$= \dfrac{\cancel{2}\sqrt{2}(\sqrt{3}+1)}{\cancel{2}}$

$= \sqrt{2}(\sqrt{3}+1)$

1. (i) $\sin(-75°)$

$= -\sin 75°$

$= -\sin(45°+30°)$

$= -(\sin 45° \cos 30° + \cos 45° \sin 30°)$

$= -\left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right)$

$= -\dfrac{\sqrt{3}+1}{2\sqrt{2}}$

2. (1) $\tan 75° = \tan(45°+30°)$

$= \dfrac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$

$= \dfrac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \dfrac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \dfrac{(\sqrt{3}+1)^2}{3-1}$

$= \dfrac{3+1+2\sqrt{3}}{2}$

$= \dfrac{4+2\sqrt{3}}{2}$

$= 2+\sqrt{3}$

$\cot 75° = \dfrac{1}{\tan 75°} = \dfrac{1}{2+\sqrt{3}} = \dfrac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = \dfrac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

$\therefore \tan 75° - \cot 75°$

$= 2+\sqrt{3} - (2-\sqrt{3})$

$= 2\sqrt{3}$

$= 4 \cdot \frac{1}{2} \cdot \sqrt{3}$

$= 4 \cdot \frac{\sqrt{3}}{2}$

$= 4 \cdot \sin 60°$

(ii) $\cos\alpha + \cos(120^\circ+\alpha) + \cos(120^\circ-\alpha)$

$= \cos\alpha + \cos 120^\circ \cos\alpha - \sin 120^\circ \sin\alpha + \cos 120^\circ \cos\alpha + \sin 120^\circ \sin\alpha$

$= \cos\alpha + 2\cos 120^\circ \cos\alpha$ $\quad \left[\because \cos 120^\circ = \cos(2\times 90^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2}\right]$

$= \cos\alpha + 2\times\left(-\frac{1}{2}\right)\cos\alpha$

$= \cos\alpha - \cos\alpha$

$= 0$

(iii) আমরা জানি,

$\tan 45^\circ = 1$

বা $\tan(35^\circ + 10^\circ) = 1$

বা $\dfrac{\tan 35^\circ + \tan 10^\circ}{1 - \tan 35^\circ \tan 10^\circ} = 1$

বা $\tan 35^\circ + \tan 10^\circ = 1 - \tan 35^\circ \tan 10^\circ$

বা $\tan 35^\circ + \tan 10^\circ + \tan 35^\circ \tan 10^\circ = 1$

(iv) $\tan 8\alpha = \tan(5\alpha + 3\alpha)$

বা, $\dfrac{\tan 8\alpha}{1} = \dfrac{\tan 5\alpha + \tan 3\alpha}{1 - \tan 5\alpha \tan 3\alpha}$

বা $\tan 8\alpha - \tan 8\alpha \tan 5\alpha \tan 3\alpha = \tan 5\alpha + \tan 3\alpha$

বা $\tan 8\alpha - \tan 5\alpha - \tan 3\alpha = \tan 8\alpha \tan 5\alpha \tan 3\alpha$

(v) $\tan 43^\circ = \tan(45^\circ - 2^\circ)$

$= \dfrac{\tan 45^\circ - \tan 2^\circ}{1 + \tan 45^\circ \tan 2^\circ}$

$= \dfrac{1 - \tan 2^\circ}{1 + \tan 2^\circ}$

$= \dfrac{1 - \frac{\sin 2^\circ}{\cos 2^\circ}}{1 + \frac{\sin 2^\circ}{\cos 2^\circ}}$

$= \dfrac{\cos 2^\circ - \sin 2^\circ}{\cos 2^\circ + \sin 2^\circ}$

(vi) $\tan 62^\circ = \tan(34^\circ + 28^\circ)$

বা $\dfrac{\tan 62^\circ}{1} = \dfrac{\tan 34^\circ + \tan 28^\circ}{1 - \tan 34^\circ \tan 28^\circ}$

বা, $\tan 62^\circ - \tan 34^\circ \tan 28^\circ \tan 62^\circ = \tan 34^\circ + \tan 28^\circ$

বা $\tan 62^\circ - \tan 34^\circ \tan 28^\circ \tan(90^\circ - 28^\circ) = \tan 34^\circ + \tan 28^\circ$

বা $\tan 62^\circ - \tan 34^\circ \tan 28^\circ \cot 28^\circ = \tan 34^\circ + \tan 28^\circ$

বা $\tan 62^\circ - \tan 34^\circ \times 1 = \tan 34^\circ + \tan 28^\circ$ $[\because \tan 28^\circ \cot 28^\circ = 1]$

বা $\tan 62^\circ = 2\tan 34^\circ + \tan 28^\circ$

3 (c) $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$$= \dfrac{\frac{2}{5} + \frac{3}{7}}{1 - \frac{2}{5} \times \frac{3}{7}}$$

$$= \dfrac{\frac{14+15}{35}}{\frac{35-6}{35}}$$

$$= \dfrac{29}{29} = 1 = \tan 45^\circ$$

$\therefore \alpha+\beta = 45^\circ$

3d (ii) $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

$$= \dfrac{\frac{4}{7} + \frac{1}{7} + \frac{1}{8} - \frac{4}{7} \times \frac{1}{7} \times \frac{1}{8}}{1 - \frac{4}{7} \times \frac{1}{7} - \frac{1}{7} \times \frac{1}{8} - \frac{1}{8} \times \frac{4}{7}}$$

$$= \dfrac{325}{325} = 1 = \tan 45^\circ$$

$\therefore A+B+C = 45^\circ$

4(i) $\dfrac{\cos 20^\circ + \sin 20^\circ}{\cos 20^\circ - \sin 20^\circ}$ লব ও হরে $\cos 20^\circ$ দ্বারা ভাগ করে

$$= \dfrac{1 + \tan 20^\circ}{1 - \tan 20^\circ}$$

$$= \dfrac{\tan 45^\circ + \tan 20^\circ}{1 - \tan 45^\circ \tan 20^\circ}$$

$$= \tan(45^\circ + 20^\circ)$$

$$= \tan 65^\circ$$

(ii) $\operatorname{cosec}(\theta+\phi) = \dfrac{1}{\sin(\theta+\phi)}$

$$= \dfrac{1}{\sin\theta\cos\phi + \cos\theta\sin\phi}$$ লব ও হরে $\sin\theta\sin\phi$ দ্বারা ভাগ করে

$$= \frac{\operatorname{cosec}\theta \operatorname{cosec}\phi}{\frac{\sin\theta\cos\phi}{\sin\theta\sin\phi} + \frac{\cos\theta\sin\phi}{\sin\theta\sin\phi}}$$

$$= \frac{\operatorname{cosec}\theta \operatorname{cosec}\phi}{\cot\phi + \cot\theta}$$

(iii) ~~$\cos 67^\circ \cos 24$~~ $\cos 67^\circ 24' \cos 7^\circ 24' + \cos 82^\circ 36' \cos 22^\circ 36'$

~~$\cos 67^\circ \cos$~~

$= \cos 67^\circ 24' \cos 7^\circ 24' + \sin(90^\circ - 82^\circ 36') \sin(90^\circ - 22^\circ 36')$

$= \cos 67^\circ 24' \cos 7^\circ 24' + \sin 7^\circ 24' \sin 67^\circ 24'$

$= \cos(67^\circ 24' - 7^\circ 24')$

$= \cos 60^\circ = \frac{1}{2}$

(iv) ধরা যাক $B - C = x,\ C - A = y,\ A - B = z$

$\therefore x + y + z = B - C + C - A + A - B = 0$

$\therefore x + y = -z$

$\therefore \tan(x+y) = \tan(-z)$

বা $\frac{\tan x + \tan y}{1 - \tan x \tan y} = -\tan z$

বা $\tan x + \tan y = -\tan z + \tan x \tan y \tan z$

বা $\tan x + \tan y + \tan z = \tan x \tan y \tan z$

x, y, z এর মান বসিয়ে পাই

$\therefore \tan(B-C) + \tan(C-A) + \tan(A-B) = \tan(B-C)\tan(C-A)\tan(A-B)$

5 (i) $\left[\cot\left(\frac{\pi}{4} - \theta\right) - 1\right](\cot\theta - 1)$

$= \left[\frac{\cot\frac{\pi}{4}\cot\theta + 1}{\cot\theta - \cot\frac{\pi}{4}} - 1\right](\cot\theta - 1)$

$= \left[\frac{1 \times \cot\theta + 1}{\cot\theta - 1} - 1\right](\cot\theta - 1)$

$= \left[\frac{\cot\theta + 1 - \cot\theta + 1}{\cot\theta - 1}\right](\cot\theta - 1)$

$= \frac{2}{(\cot\theta - 1)} \times (\cot\theta - 1)$

$= 2$ যা θ নিরপেক্ষ

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6.

$\cos(\alpha-\beta)+1=0$

বা $\cos\alpha\cos\beta+\sin\alpha\sin\beta+1=0$

বা $2\cos\alpha\cos\beta+2\sin\alpha\sin\beta+2=0$ [উভয় পক্ষে 2 দ্বারা গুণ করে।]

বা $2\cos\alpha\cos\beta+2\sin\alpha\sin\beta+1+1=0$

বা $2\cos\alpha\cos\beta+2\sin\alpha\sin\beta+\sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta=0$

বা $\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta+\sin^2\alpha+\sin^2\beta+2\sin\alpha\sin\beta=0$

বা $(\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2=0$

দুটি বাস্তব রাশির বর্গের সমষ্টি শূন্য হলে প্রত্যেক প্রত্যেকটি শূন্য হবে

$\therefore \cos\alpha+\cos\beta=0$ এবং $\sin\alpha+\sin\beta=0$

<u>মডিউল এক্সারসাইজ</u>

1 (i) $\sin^2\alpha+\sin^2(120^\circ-\alpha)+\sin^2(120^\circ+\alpha)$

$=\sin^2\alpha+(\sin120^\circ\cos\alpha-\cos120^\circ\sin\alpha)^2+(\sin120^\circ\cos\alpha+\cos120^\circ\sin\alpha)^2$

$=\sin^2\alpha+2\{(\sin120^\circ\cos\alpha)^2+(\cos120^\circ\sin\alpha)^2\}$

$=\sin^2\alpha+2\{(\frac{\sqrt3}{2}\cos\alpha)^2+(-\frac12\sin\alpha)^2\}$

$=\sin^2\alpha+2(\frac34\cos^2\alpha+\frac14\sin^2\alpha)$

$=\sin^2\alpha+\frac32\cos^2\alpha+\frac12\sin^2\alpha$

$=\frac32\sin^2\alpha+\frac32\cos^2\alpha$

$=\frac32(\sin^2\alpha+\cos^2\alpha)$

$=\frac32\times1$

$=\frac32$

[$\because \sin120^\circ=\sin(2\times90^\circ-60^\circ)=\sin60^\circ=\frac{\sqrt3}{2}$ এবং $\cos120^\circ=-\frac12$]

(ii) $\sin^2(\frac{\pi}{8}+\frac{\theta}{2})-\sin^2(\frac{\pi}{8}-\frac{\theta}{2})$

$=\sin(\frac{\pi}{8}+\frac{\theta}{2}+\frac{\pi}{8}-\theta/2)\sin(\frac{\pi}{8}+\frac{\theta}{2}-\frac{\pi}{8}+\theta/2)$

$=\sin\frac{\pi}{4}\cdot\sin\theta=\frac{1}{\sqrt2}\sin\theta$

(iii) $\cos^2 A + \cos^2\left(A+\frac{\pi}{3}\right) + \cos^2\left(A-\frac{\pi}{3}\right)$

$= \cos^2 A + \left(\cos A\cos\frac{\pi}{3} - \sin A\sin\frac{\pi}{3}\right)^2 + \left(\cos A\cos\frac{\pi}{3} + \sin A\sin\frac{\pi}{3}\right)^2$

$= \cos^2 A + 2\left\{\left(\cos A\cos\frac{\pi}{3}\right)^2 + \left(\sin A\sin\frac{\pi}{3}\right)^2\right\}$

$= \cos^2 A + 2\left\{\left(\frac{1}{2}\cos A\right)^2 + \left(\frac{\sqrt{3}}{2}\sin A\right)^2\right\}$

$= \cos^2 A + 2\left(\frac{1}{4}\cos^2 A + \frac{3}{4}\sin^2 A\right)$

$= \cos^2 A + \frac{1}{2}\cos^2 A + \frac{3}{2}\sin^2 A$

$= \frac{3}{2}\cos^2 A + \frac{3}{2}\sin^2 A$

$= \frac{3}{2}\left(\cos^2 A + \sin^2 A\right)$

$= \frac{3}{2}$

(iv) $\tan 70^\circ = \tan(50^\circ + 20^\circ)$

or $\frac{\tan 70^\circ}{1} = \frac{\tan 50^\circ + \tan 20^\circ}{1 - \tan 50^\circ \tan 20^\circ}$

or $\tan 70^\circ - \tan 70^\circ \tan 50^\circ \tan 20^\circ = \tan 50^\circ + \tan 20^\circ$

or $\tan 70^\circ - \tan(90^\circ - 20^\circ)\tan 50^\circ \tan 20^\circ = \tan 50^\circ + \tan 20^\circ$

or $\tan 70^\circ - \cot 20^\circ \tan 20^\circ \tan 50^\circ = \tan 50^\circ + \tan 20^\circ$

or $\tan 70^\circ - \tan 50^\circ = \tan 50^\circ + \tan 20^\circ$

$\therefore \tan 70^\circ = 2\tan 50^\circ + \tan 20^\circ$

2. (iii) $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{3\pi}{4}+\theta\right)$

$= \frac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta} \cdot \frac{\tan\frac{3\pi}{4} + \tan\theta}{1 - \tan\frac{3\pi}{4}\cdot\tan\theta}$

$\left[\because \tan\frac{3\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\left(2\cdot\frac{\pi}{2} - \frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1\right]$

$= \frac{1+\tan\theta}{1-\tan\theta} \cdot \frac{(-1+\tan\theta)}{1-(-1)\tan\theta}$

$= -\frac{(1+\tan\theta)(1-\tan\theta)}{(1-\tan\theta)(1+\tan\theta)}$

$= -1$

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2 (iv) $\sin(B+C)\sin(B-C) + \sin(C+A)\sin(C-A) + \sin(A+B)\sin(A-B)$

$= \sin^2 B - \sin^2 C + \sin^2 C - \sin^2 A + \sin^2 A - \sin^2 B$

$= 0$

4.

$\sin\alpha \sin\beta - \cos\alpha\cos\beta + 1 = 0$

$\Rightarrow \cos\alpha\cos\beta - \sin\alpha\sin\beta = 1$

$\Rightarrow \cos(\alpha+\beta) = 1$

আবার, $\sin^2(\alpha+\beta) = 1 - \cos^2(\alpha+\beta)$

$= 1-1$

$= 0$

$\therefore \sin(\alpha+\beta) = 0$

আবার,

$1 + \cot\alpha\tan\beta$

$= 1 + \frac{\cos\alpha}{\sin\alpha}\frac{\sin\beta}{\cos\beta}$

$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta}$

$= \frac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta}$ $[\because \sin(\alpha+\beta) = 0]$

$= \frac{0}{\sin\alpha\cos\beta}$

$= 0$

5.

$A + B = 45^\circ$ —— (1)

$\Rightarrow A = 45^\circ - B$

$\therefore \tan A = \tan(45^\circ - B)$

$\Rightarrow \tan A = \frac{\tan 45^\circ - \tan B}{1 + \tan 45^\circ \tan B}$

$\Rightarrow \tan A = \frac{1 - \tan B}{1 + \tan B}$ {উভয়পক্ষে 1 যোগ করে}

$\Rightarrow 1 + \tan A = 1 + \frac{1 - \tan B}{1 + \tan B}$

$= \frac{1 + \tan B + 1 - \tan B}{1 + \tan B}$

$(1+\tan A)(1+\tan B) = 2$ —— (ii)

এখানে, $B = A$

(i) নং থেকে পাই $2A = 45°$

বা $A = 22\frac{1}{2}°$

(ii) নং $B = A$ বসিয়ে পাই

$(1+\tan A)(1+\tan A) = 2$

বা $(1+\tan A)^2 = 2$

বা $1+\tan A = \pm\sqrt{2}$

বা $\tan A = \pm\sqrt{2} - 1$

$\because A = 22\frac{1}{2}°$

$\therefore \tan A = \tan 22\frac{1}{2}° > 0$

$\therefore \tan A = \tan 22\frac{1}{2}° = \sqrt{2} - 1$

6.

$\tan A = \dfrac{\sin A}{\cos A}$

$= \dfrac{\sin\{\pi - (B+C)\}}{\cos B \cos C}$ $\quad \because A+B+C = \pi \quad \therefore B+C = \pi - A$

$= \dfrac{\sin\{2\cdot\frac{\pi}{2} - (B+C)\}}{\cos B\cos C}$

$= \dfrac{\sin(B+C)}{\cos B \cos C}$

$= \dfrac{\sin B\cos C + \cos B \sin C}{\cos B \cos C}$

$= \dfrac{\sin B \cos C}{\cos B \cos C} + \dfrac{\cos B \sin C}{\cos B \cos C}$

$= \tan B + \tan C$

(ii) দেওয়া, $\cos A = \cos B \cos C$

বা $\cos\{\pi - (B+C)\} = \cos B \cos C$

বা $\cos\{2\cdot\frac{\pi}{2} - (B+C)\} = \cos B \cos C$

বা $-\cos(B+C) = \cos B \cos C$

বা $-(\cos B \cos C - \sin B \sin C) = \cos B \cos C$

বা $-\cos B\cos C + \sin B\sin C = \cos B\cos C$

বা $-2\cos B\cos C = -\sin B\sin C$

বা $\dfrac{2\cos B\cos C}{\sin B\sin C} = 1$

বা $2\cot B\cot C = 1$

7.

$\theta = \alpha + \beta$

প্রশ্নমতে, $\dfrac{\tan\alpha}{\tan\beta} = \dfrac{x}{y}$

বা, $\dfrac{\sin\alpha\cos\beta}{\cos\alpha\sin\beta} = \dfrac{x}{y}$

যোজন বিয়োজন প্রক্রিয়া করে

$\therefore \dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\alpha\cos\beta + \cos\alpha\sin\beta} = \dfrac{x-y}{x+y}$

বা $\dfrac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)} = \dfrac{x-y}{x+y}$

বা $\dfrac{\sin(\alpha-\beta)}{\sin\theta} = \dfrac{x-y}{x+y}$ $\quad \because \alpha+\beta = \theta$

বা $\sin(\alpha-\beta) = \dfrac{x-y}{x+y}\cdot\sin\theta$

8.

$\tan(\alpha-\theta) = \dfrac{\tan\alpha - \tan\theta}{1+\tan\alpha\tan\theta}$

$= \dfrac{\tan\alpha - \dfrac{q\sin\alpha}{p+q\cos\alpha}}{1+\tan\alpha\cdot\dfrac{q\sin\alpha}{p+q\cos\alpha}}$

$= \dfrac{\dfrac{\sin\alpha}{\cos\alpha} - \dfrac{q\sin\alpha}{p+q\cos\alpha}}{1+\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{q\sin\alpha}{p+q\cos\alpha}}$

$= \dfrac{\dfrac{\sin\alpha(p+q\cos\alpha) - q\sin\alpha\cos\alpha}{\cos\alpha(p+q\cos\alpha)}}{\dfrac{\cos\alpha(p+q\cos\alpha) + q\sin^2\alpha}{\cos\alpha(p+q\cos\alpha)}}$

$= \dfrac{p\sin\alpha + q\cos\alpha\sin\alpha - q\sin\alpha\cos\alpha}{p\cos\alpha + q\cos^2\alpha + q\sin^2\alpha}$

$= \dfrac{p\sin\alpha}{p\cos\alpha + q(\cos^2\alpha + \sin^2\alpha)}$

$= \dfrac{p\sin\alpha}{p\cos\alpha + q}$

9. $\sin(\alpha+\beta) = n\sin(\alpha-\beta)$

or $\sin\alpha\cos\beta + \cos\alpha\sin\beta = n(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$

or $\cos\alpha\sin\beta + n\cos\alpha\sin\beta = n\sin\alpha\cos\beta - \sin\alpha\cos\beta$

or $(n+1)\cos\alpha\sin\beta = (n-1)\sin\alpha\cos\beta$

or $\dfrac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta} = \dfrac{n-1}{n+1}$

or $\cot\alpha\tan\beta = \dfrac{n-1}{n+1}$

or $\cot\alpha = \dfrac{n-1}{(n+1)\tan\beta} = \dfrac{n-1}{n+1}\cot\beta$

10.
(i) $\cot\alpha\cot\beta = 3$

or $\dfrac{\cos\alpha}{\sin\alpha}\cdot\dfrac{\cos\beta}{\sin\beta} = \dfrac{3}{1}$

or $\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} = \dfrac{3}{1}$

(যোগ-ভাগ প্রক্রিয়া প্রয়োগ করে)

$\dfrac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} = \dfrac{3+1}{3-1}$

or $\dfrac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} = \dfrac{4}{2}$

or $\dfrac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} = 2$

(iii)
$\cos\theta + \sin\theta = \sqrt{2}$

or $\dfrac{1}{\sqrt{2}}\cos\theta + \dfrac{1}{\sqrt{2}}\sin\theta = 1$

or $\cos\theta\cos45^\circ + \sin\theta\sin45^\circ = 1$

or $\cos(\theta - 45^\circ) = 1 = \cos 0^\circ$

$\therefore \theta - 45^\circ = 0^\circ$

or $\theta = 45^\circ$

$\cos3\theta = \cos(3\times45^\circ) = \cos135^\circ$

$= \cos(2\times90^\circ - 45^\circ)$

$= -\cos45^\circ$

$= -\dfrac{1}{\sqrt{2}}$

10(ii) $\tan\alpha = 2\tan\beta$

বা $\frac{\sin\alpha}{\cos\alpha} = 2\cdot\frac{\sin\beta}{\cos\beta}$

বা $\frac{\sin\alpha\cos\beta}{\cos\alpha\sin\beta} = \frac{2}{1}$

যোজন-বিয়োজন প্রক্রিয়া প্রয়োগ করে

$$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta - \cos\alpha\sin\beta} = \frac{2+1}{2-1}$$

বা $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{3}{1} = 3$

11.

$\tan\alpha = \frac{x\sin\beta}{1 - x\cos\beta}$

বা $\frac{\sin\alpha}{\cos\alpha} = \frac{x\sin\beta}{1 - x\cos\beta}$

বা $x\sin\beta\cos\alpha = \sin\alpha - x\sin\alpha\cos\beta$

বা $x\sin\alpha\cos\beta + x\sin\beta\cos\alpha = \sin\alpha$

বা $x(\sin\alpha\cos\beta + \sin\beta\cos\alpha) = \sin\alpha$

বা $x\sin(\alpha+\beta) = \sin\alpha$ —— (i)

আবার, $\tan\beta = \frac{y\sin\alpha}{1 - y\cos\alpha}$

বা $\frac{\sin\beta}{\cos\beta} = \frac{y\sin\alpha}{1 - y\cos\alpha}$

বা $y\sin\alpha\cos\beta = \sin\beta - y\sin\beta\cos\alpha$

$y(\sin\alpha\cos\beta + \sin\beta\cos\alpha) = \sin\beta$

বা $y\sin(\alpha+\beta) = \sin\beta$ —— (ii)

(i) ÷ (ii) প্রয়োগ করে

$$\frac{x\sin(\alpha+\beta)}{y\sin(\alpha+\beta)} = \frac{\sin\alpha}{\sin\beta}$$

বা $\frac{x}{y} = \frac{\sin\alpha}{\sin\beta}$

বা $\frac{\sin\alpha}{\sin\beta} = \frac{x}{y}$

14. $\cos(\theta-\alpha) = p$

$\Rightarrow \cos\theta\cos\alpha + \sin\theta\sin\alpha = p \quad \text{--- (i)}$

আবার, $\sin(\theta+\beta) = q$

$\Rightarrow \sin\theta\cos\beta + \cos\theta\sin\beta = q \quad \text{--- (ii)}$

$\cos(\alpha+\beta) = \cos\{(\theta+\beta)-(\theta-\alpha)\}$

$= \cos(\theta+\beta)\cos(\theta-\alpha) + \sin(\theta+\beta)\sin(\theta-\alpha)$ বর্গ করে

$\therefore \cos^2(\alpha+\beta) = \cos^2(\theta+\beta)\cos^2(\theta-\alpha) + \sin^2(\theta+\beta)\sin^2(\theta-\alpha)$
$+ 2\cos(\theta+\beta)\cos(\theta-\alpha)\cdot\sin(\theta+\beta)\sin(\theta-\alpha)$

$= (1-q^2)p^2 + q^2(1-p^2) + 2pq\sin(\theta-\alpha)\cos(\theta+\beta)$

$= p^2 - p^2q^2 + q^2 - p^2q^2 + 2pq\sin(\theta-\alpha)\cos(\theta+\beta)$

$= p^2+q^2 - 2p^2q^2 + 2pq\sin(\theta-\alpha)\cos(\theta+\beta)$

$= p^2+q^2 - 2pq\{pq - \sin(\theta-\alpha)\cos(\theta+\beta)\}$

$= p^2+q^2 - 2pq\{\cos(\theta-\alpha)\sin(\theta+\beta) - \sin(\theta-\alpha)\cos(\theta+\beta)\}$

$= p^2+q^2 - 2pq\sin(\theta+\beta-\theta+\alpha)$

$= p^2+q^2 - 2pq\sin(\alpha+\beta)$

12. $x = \tan\theta + \tan\phi$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\sin\phi}{\cos\phi}$

$= \dfrac{\sin\theta\cos\phi + \cos\theta\sin\phi}{\cos\theta\cos\phi}$

$= \dfrac{\sin(\theta+\phi)}{\cos\theta\cos\phi}$

$\therefore \dfrac{1}{x} = \dfrac{\cos\theta\cos\phi}{\sin(\theta+\phi)}$

আবার, $y = \cot\theta + \cot\phi$

$= \dfrac{\cos\theta}{\sin\theta} + \dfrac{\cos\phi}{\sin\phi}$

$= \dfrac{\sin\phi\cos\theta + \sin\theta\cos\phi}{\sin\theta\sin\phi} = \dfrac{\sin(\theta+\phi)}{\sin\theta\sin\phi}$

$\Rightarrow \dfrac{1}{y} = \dfrac{\sin\theta\sin\phi}{\sin(\theta+\phi)}$

$\therefore \dfrac{1}{x} - \dfrac{1}{y} = \dfrac{\cos\theta\cos\phi}{\sin(\theta+\phi)} - \dfrac{\sin\theta\sin\phi}{\sin(\theta+\phi)}$

$= \dfrac{\cos\theta\cos\phi - \sin\theta\sin\phi}{\sin(\theta+\phi)}$

$= \dfrac{\cos(\theta+\phi)}{\sin(\theta+\phi)} = \cot(\theta+\phi)$

13. $\tan(\theta+30^\circ) = \dfrac{\tan\theta + \tan 30^\circ}{1-\tan\theta \tan 30^\circ}$

$= \dfrac{\tan\theta + \frac{1}{\sqrt{3}}}{1-\tan\theta\cdot\frac{1}{\sqrt{3}}}$

$= \dfrac{\frac{2x-K}{K\sqrt{3}} + \frac{1}{\sqrt{3}}}{1-\frac{2x-K}{K\sqrt{3}}\cdot\frac{1}{\sqrt{3}}}$

$= \dfrac{\frac{2x-K+K}{K\sqrt{3}}}{\frac{3K-(2x-K)}{(K\sqrt{3})\sqrt{3}}}$

$= \dfrac{2x}{3K-2x+K} \times \dfrac{3K}{K\sqrt{3}}$

$= \dfrac{2x}{4K-2x} \times \sqrt{3}$

$= \dfrac{x\sqrt{3}}{2K-x} = \tan\varphi$

$\therefore \theta+30^\circ = \varphi$

or $\theta-\varphi = -30^\circ$

or $|\theta-\varphi| = |-30^\circ| = 30^\circ$

[illegible] 30°

1. $\tan(A+B) - \tan(A-B)$

$= \dfrac{\sin(A+B)}{\cos(A+B)} - \dfrac{\sin(A-B)}{\cos(A-B)}$

$= \dfrac{\sin(A+B)\cos(A-B) - \cos(A+B)\sin(A-B)}{\cos(A+B)\cos(A-B)}$

$= \dfrac{\sin(A+B-A+B)}{\cos^2 B - \sin^2 A}$

$= \dfrac{\sin 2B}{\cos^2 B - \sin^2 A}$

2. $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

$= \dfrac{\frac{\sin\alpha}{\cos\alpha} - \frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}}{1+\frac{\sin\alpha}{\cos\alpha}\cdot\frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}}$

$= \dfrac{\frac{\sin\alpha(1-n\sin^2\alpha) - n\sin\alpha\cos^2\alpha}{\cos\alpha(1-n\sin^2\alpha)}}{\frac{\cos\alpha(1-n\sin^2\alpha) + n\sin^2\alpha\cos\alpha}{\cos\alpha(1-n\sin^2\alpha)}}$

$$= \frac{\sin\alpha - n\sin^3\alpha - n\sin\alpha\cos^2\alpha}{\cos\alpha - n\sin^2\alpha\cos\alpha + n\sin^2\alpha\cos\alpha}$$

$$= \frac{\sin\alpha - n\sin\alpha\{\sin^2\alpha + \cos^2\alpha\}}{\cos\alpha}$$

$$= \frac{\sin\alpha - n\sin\alpha}{\cos\alpha}$$

$$= \frac{(1-n)\sin\alpha}{\cos\alpha}$$

$$= (1-n)\tan\alpha$$

3. $m\tan(\theta - 30^\circ) = n\tan(\theta + 120^\circ)$

বা, $\frac{m}{n} = \frac{\tan(\theta+120^\circ)}{\tan(\theta-30^\circ)}$

বা, $\frac{m}{n} = \frac{\sin(\theta+120^\circ)\cos(\theta-30^\circ)}{\cos(\theta+120^\circ)\sin(\theta-30^\circ)}$

(যোজন বিয়োজন প্রক্রিয়া করে)

$$\frac{m+n}{m-n} = \frac{\sin(\theta+120^\circ)\cos(\theta-30^\circ) + \cos(\theta+120^\circ)\sin(\theta-30^\circ)}{\sin(\theta+120^\circ)\cos(\theta-30^\circ) - \cos(\theta+120^\circ)\sin(\theta-30^\circ)}$$

$$= \frac{\sin(\theta+120^\circ+\theta-30^\circ)}{\sin(\theta+120^\circ-\theta+30^\circ)}$$

$$= \frac{\sin(2\theta+90^\circ)}{\sin 150^\circ} \qquad \because \sin 150^\circ = \sin(2\times 90^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}$$

$$= \frac{\sin(90^\circ+2\theta)}{\frac{1}{2}}$$

$$= 2\cos 2\theta$$

$\therefore 2\cos 2\theta = \frac{m+n}{m-n}$

4. $\frac{\sin(\theta+\phi)}{\cos(\theta-\phi)} = \frac{1-x}{1+x}$

(যোজন বিয়োজন প্রক্রিয়া করে)

$$\frac{\sin(\theta+\phi)+\cos(\theta-\phi)}{\sin(\theta+\phi)-\cos(\theta-\phi)} = \frac{1-x+1+x}{1-x-1-x} = \frac{2}{-2x} = -\frac{1}{x}$$

বা $$\frac{\sin\theta\cos\phi + \cos\theta\sin\phi + \cos\theta\cos\phi + \sin\theta\sin\phi}{\sin\theta\cos\phi + \cos\theta\sin\phi - \cos\theta\cos\phi - \sin\theta\sin\phi} = -\frac{1}{x}$$

বা $$\frac{\sin\theta\cos\phi + \cos\theta\cos\phi + \cos\theta\sin\phi + \sin\theta\sin\phi}{\sin\theta\cos\phi - \cos\theta\cos\phi - \sin\theta\sin\phi + \cos\theta\sin\phi}$$

বা $$\frac{\cos\phi(\sin\theta+\cos\theta) + \sin\phi(\cos\theta+\sin\theta)}{\cos\phi(\sin\theta-\cos\theta) - \sin\phi(\sin\theta-\cos\theta)} = -\frac{1}{x}$$

বা $\frac{(\sin\theta+\cos\theta)(\cos\varphi+\sin\varphi)}{(\sin\theta-\cos\theta)(\cos\varphi-\sin\varphi)} = -\frac{1}{x}$ Page-15 এখানে সমাধান দেখো নাই

বা $\frac{(\sin\theta-\cos\theta)(\cos\varphi-\sin\varphi)}{(\sin\theta+\cos\theta)(\cos\varphi+\sin\varphi)} = -x$

বা $\frac{(\cos\theta-\sin\theta)}{\cos\theta+\sin\theta}\cdot\frac{\cos\varphi-\sin\varphi}{\cos\varphi+\sin\varphi} = x$

বা $\frac{1-\tan\theta}{1+\tan\theta}\cdot\frac{1-\tan\varphi}{1+\tan\varphi} = x$

বা $\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\cdot\frac{\tan\frac{\pi}{4}-\tan\varphi}{1+\tan\frac{\pi}{4}\tan\varphi} = x$

বা $\tan\left(\frac{\pi}{4}-\theta\right)\cdot\tan\left(\frac{\pi}{4}-\varphi\right) = x$

5. $\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta) = -3/2$

বা $2\{\cos\beta\cos\gamma+\sin\beta\sin\gamma+\cos\gamma\cos\alpha+\sin\gamma\sin\alpha+\cos\alpha\cos\beta+\sin\alpha\sin\beta\} = -3$

বা $2\cos\beta\cos\gamma+2\sin\beta\sin\gamma+2\cos\gamma\cos\alpha+2\sin\gamma\sin\alpha+2\cos\alpha\cos\beta+2\sin\alpha\sin\beta+1+1+1=0$

বা $2\cos\beta\cos\gamma+2\sin\beta\sin\gamma+2\cos\gamma\cos\alpha+2\sin\gamma\sin\alpha+2\cos\alpha\cos\beta+2\sin\alpha\sin\beta+\sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta+\sin^2\gamma+\cos^2\gamma=0$

বা, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+2\cos\alpha\cos\beta+2\cos\gamma\cos\alpha+2\cos\beta\cos\gamma+\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2\sin\alpha\sin\beta+2\sin\beta\sin\gamma+2\sin\gamma\sin\alpha=0$

বা $(\cos\alpha+\cos\beta+\cos\gamma)^2+(\sin\alpha+\sin\beta+\sin\gamma)^2=0$ যেহেতু দুটি বর্গের যোগফল শূন্য তাই প্রত্যেকটি পৃথকভাবে শূন্য হবে

$\therefore \cos\alpha+\cos\beta+\cos\gamma=0$ এবং $\sin\alpha+\sin\beta+\sin\gamma=0$

এখন যেহেতু $\cos\beta+\cos\gamma+\cos\alpha=0$

বা $\cos\beta+\cos\gamma=-\cos\alpha$

বর্গ করে

$\cos^2\beta+\cos^2\gamma+2\cos\beta\cos\gamma=\cos^2\alpha$ — (i)

আবার $\sin\beta+\sin\gamma+\sin\alpha=0$

বা, $\sin\beta+\sin\gamma=-\sin\alpha$

বর্গ করে

$\sin^2\beta+\sin^2\gamma+2\sin\beta\sin\gamma=\sin^2\alpha$ — (ii)

(i) ও (ii) যোগ করে

$\cos^2\beta+\cos^2\gamma+2\cos\beta\cos\gamma+\sin^2\beta+\sin^2\gamma+2\sin\beta\sin\gamma=\cos^2\alpha+\sin^2\alpha$

$\cos^2\beta+\cos^2\gamma+2\cos\beta\cos\gamma+2\sin\beta\sin\gamma=1$

বা $\cos^2\beta+\sin^2\beta+\cos^2\gamma+\sin^2\gamma+2\cos\beta\cos\gamma+2\sin\beta\sin\gamma=1$

$\Rightarrow 1 + 1 + 2\cos(\beta - \gamma) = 1$

$\Rightarrow 2\cos(\beta - \gamma) = 1 - 2$

$\Rightarrow \cos(\beta - \gamma) = -\frac{1}{2}$

একইভাবে প্রমাণ করা যায় $\cos(\gamma - \alpha) = -\frac{1}{2}$

এবং $\cos(\alpha - \beta) = -\frac{1}{2}$

$\therefore \cos(\beta - \gamma) = \cos(\gamma - \alpha) = \cos(\alpha - \beta) = -\frac{1}{2}$

6.

$a\tan\alpha + b\tan\beta = (a+b)\tan\left(\frac{\alpha+\beta}{2}\right)$

$\Rightarrow a\tan\alpha - a\tan\frac{\alpha+\beta}{2} = b\tan\frac{\alpha+\beta}{2} - b\tan\beta$

$$\Rightarrow a\left\{\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha+\beta}{2}}\right\} = b\left\{\frac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha+\beta}{2}} - \frac{\sin\beta}{\cos\beta}\right\}$$

$$\Rightarrow a\,\frac{\left\{\sin\alpha\cos\frac{\alpha+\beta}{2} - \cos\alpha\sin\frac{\alpha+\beta}{2}\right\}}{\cos\alpha\cos\frac{\alpha+\beta}{2}} = b\,\frac{\left\{\sin\frac{\alpha+\beta}{2}\cos\beta - \cos\frac{\alpha+\beta}{2}\sin\beta\right\}}{\cos\frac{\alpha+\beta}{2}\cos\beta}$$

$$\Rightarrow a\,\frac{\sin\left(\alpha - \frac{\alpha+\beta}{2}\right)}{\cos\alpha} = b\,\frac{\sin\left(\frac{\alpha+\beta}{2} - \beta\right)}{\cos\beta}$$

$$\Rightarrow \frac{a\left(\sin\frac{\alpha-\beta}{2}\right)}{\cos\alpha} = \frac{b\left(\sin\frac{\alpha-\beta}{2}\right)}{\cos\beta}$$

$$\Rightarrow \frac{\cos\alpha}{\cos\beta} = \frac{a}{b}$$

7.

$\sin\theta = K\sin(\theta + \phi)$

$\Rightarrow \sin(\theta + \phi - \phi) = K\sin(\theta + \phi)$

$\Rightarrow \sin(\theta+\phi)\cos\phi - \cos(\theta+\phi)\sin\phi = K\sin(\theta+\phi)$

$\Rightarrow \sin(\theta+\phi)\cos\phi - K\sin(\theta+\phi) = \cos(\theta+\phi)\sin\phi$

$\Rightarrow \sin(\theta+\phi)\{\cos\phi - K\} = \cos(\theta+\phi)\sin\phi$

$$\Rightarrow \frac{\sin(\theta+\phi)}{\cos(\theta+\phi)} = \frac{\sin\phi}{\cos\phi - K}$$

$$\Rightarrow \tan(\theta+\phi) = \frac{\sin\phi}{\cos\phi - K}$$

8.
$$\frac{\cot(\alpha-\beta)}{\cot\alpha} + \frac{\cos^2\gamma}{\cos^2\alpha} = 1$$

or, $\dfrac{\cos^2\gamma}{\cos^2\alpha} = 1 - \dfrac{\cot(\alpha-\beta)}{\cot\alpha}$

or, " $= 1 - \dfrac{\cos(\alpha-\beta)\sin\alpha}{\sin(\alpha-\beta)\cos\alpha}$

or, " $= \dfrac{\sin(\alpha-\beta)\cos\alpha - \cos(\alpha-\beta)\sin\alpha}{\sin(\alpha-\beta)\cos\alpha}$

or, " $= \dfrac{\sin(\alpha-\beta-\alpha)}{\sin(\alpha-\beta)\cos\alpha}$

or, $\dfrac{\cos^2\gamma}{\cos^2\alpha} = -\dfrac{\sin\beta}{\sin(\alpha-\beta)\cos\alpha}$

$\therefore \cos^2\gamma = -\dfrac{\sin\beta \times \cos^2\alpha}{\sin(\alpha-\beta)\cos\alpha} = -\dfrac{\sin\beta\cos\alpha}{\sin(\alpha-\beta)}$

$\therefore \sin^2\gamma = 1 - \cos^2\gamma$

$= 1 + \dfrac{\sin\beta\cos\alpha}{\sin(\alpha-\beta)}$

$= \dfrac{\sin(\alpha-\beta) + \sin\beta\cos\alpha}{\sin(\alpha-\beta)}$

$= \dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta + \sin\beta\cos\alpha}{\sin(\alpha-\beta)}$

$= \dfrac{\sin\alpha\cos\beta}{\sin(\alpha-\beta)}$

$\therefore \tan^2\gamma = \dfrac{\sin^2\gamma}{\cos^2\gamma}$

$= \dfrac{\sin\alpha\cos\beta}{\sin(\alpha-\beta)} \cdot \left\{-\dfrac{\sin(\alpha-\beta)}{\sin\beta\cos\alpha}\right\}$

$= -\dfrac{\sin\alpha}{\cos\alpha} \cdot \dfrac{\cos\beta}{\sin\beta} \cdot \dfrac{\sin(\alpha-\beta)}{\sin(\alpha-\beta)}$

$= -\tan\alpha\cot\beta$

or $\tan^2\gamma + \tan\alpha\cot\beta = 0$

9.
$$\tan\theta = \frac{x\sin\alpha + y\sin\beta}{x\cos\alpha + y\cos\beta}$$

or $\dfrac{\sin\theta}{\cos\theta} = \dfrac{x\sin\alpha + y\sin\beta}{x\cos\alpha + y\cos\beta}$

or $x\sin\theta\cos\alpha + y\sin\theta\cos\beta = x\cos\theta\sin\alpha + y\cos\theta\sin\beta$

or $x(\sin\theta\cos\alpha - \cos\theta\sin\alpha) + y(\sin\theta\cos\beta - \cos\theta\sin\beta) = 0$

or $x\sin(\theta-\alpha) + y\sin(\theta-\beta) = 0$

10 (iii) $5\cos\theta + 12\sin\theta + 12$

[ধরা যাক $5 = r\sin\alpha$, $12 = r\cos\alpha$,

$\therefore 5^2 + 12^2 = r^2(\sin^2\alpha + \cos^2\alpha)$

বা $r^2 = 25 + 144 = 169$

বা $r = 13$]

$\therefore r\sin\alpha\cos\theta + r\cos\alpha\sin\theta + 12$

$= r\sin(\alpha+\theta) + 12$

$= 13\sin(\alpha+\theta) + 12$

আমরা জানি, $-1 \le \sin(\alpha+\theta) \le 1$

বা $-13 \le 13\sin(\alpha+\theta) \le 13$

বা $-13+12 \le 13\sin(\alpha+\theta) + 12 \le 13+12$

বা $-1 \le 13\sin(\alpha+\theta) + 12 \le 25$

$\therefore$ সর্বোচ্চ ও সর্বনিম্ন মান যথাক্রমে 25 এবং -1

(iv) $a\cos\theta + b\sin(\theta+\alpha)$

$= a\cos\theta + b(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$

$= a\cos\theta + b\cos\theta\sin\alpha + b\sin\theta\cos\alpha$

$= \cos\theta(a + b\sin\alpha) + (b\cos\alpha)\sin\theta$

[ধরা যাক $a + b\sin\alpha = A$, $b\cos\alpha = B$]

$= A\cos\theta + B\sin\theta$ —— ①

ধরা যাক $A = r\sin\beta$, $B = r\cos\beta$

$\therefore A^2 + B^2 = r^2(\sin^2\beta + \cos^2\beta) = r^2$

বা $r = \sqrt{A^2+B^2}$

$= \sqrt{(a+b\sin\alpha)^2 + (b\cos\alpha)^2}$

$= \sqrt{a^2 + 2ab\sin\alpha + b^2\sin^2\alpha + b^2\cos^2\alpha}$

$= \sqrt{a^2 + 2ab\sin\alpha + b^2(\sin^2\alpha + \cos^2\alpha)}$

$= \sqrt{a^2 + 2ab\sin\alpha + b^2}$

$\therefore \sqrt{a^2 + 2ab\sin\alpha + b^2}$

① নং থেকে পাই

$\therefore a\cos\theta + b\sin(\theta+\alpha)$

$= A\cos\theta + B\sin\theta$

$= r\sin\beta\cos\theta + r\cos\beta\sin\theta$

$= r(\sin\beta\cos\theta + \cos\beta\sin\theta)$

$= r\sin(\beta+\theta)$

আমরা জানি $-1 \le \sin(\beta+\theta) \le 1$

বা $-r \le r\sin(\beta+\theta) \le r$

$\therefore$ বৃহত্তম সর্বোচ্চ মান $= r = \sqrt{a^2 + 2ab\sin\alpha + b^2}$